The Perturbations of Halley's Comet in the Past. Fifth Paper. The period B.C. 240 *to* A.D. 760. By P. H. Cowell and A. C. D. Crommelin.

We have once more to commence by acknowledgment of the continued assistance that we have received from Dr. Smart and Mr. Cripps in the calculation of the mechanical quadratures.

The fourth paper of this series (*M.N.*, May 1908) traced the comet back to A.D. 760 June 10 (Julian day 1998810), the corrected value of n at that epoch being 46″·113. Hind's date for the previous passage is 684 Oct. 18, though the observed positions are too vague to fix the time of perihelion within several days. Mr. Knobel has shown that this comet was also observed in Japan (*M.N.*, lxvi., 2, p. 72) the date of first apparition, 684 Sept. 7, being in good accord with the date Sept. 6 of the Chinese records; he has, however, by a slip, given Hind's date of perihelion passage as Sept. 18 instead of Oct. 18.

Our computations indicate that Hind's identification is correct, the calculated date being 39 days later than his, which is a reasonable discordance for that remote epoch.

Revolution 684–760.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0°–30°	−″·002	″...	−″ 56
,,	330–360	−·006	...	...
Earth	0–30	+·011	...	+ 308
,,	330–360	−·002	...	...
Jupiter	0–90	+1·137	+560	+31720
,,	90–270	+·148	−523	+ 636
,,	270–360	−·983	+ 48	+ 170
Saturn	0–90	+·0219	+ 34	+ 612
,,	90–270	−·0312	−129	− 466
,,	270–360	+·0381	+ 54	− 7
Uranus	0–360	+·012	...	+ 280
	Sums	+·344	+ 44	+33197

Taking n in 760 as 46″·113, n in 684 = 45″·769, and period in days $= \frac{1296000 - 33197}{45 \cdot 769} = 27591$, which brings us to J.D. 1971219 = 684 Nov. 26.

The observations will scarcely permit so late a day as this for the perihelion passage. If we take the mean of Hind's value (Oct. 18) and our value Nov. 26 as the actual date, we must take n in 684 as 45″·737.

Proceeding to the revolution before this, an approximate computation quickly showed that Hind's date 608 Oct. 19 was about $1\frac{1}{2}$ year too late. Dr. Ångström had already deduced from his empirical curve that "l'apparition pour l'année 608, présentant l'écart le plus grand, est probablement douteuse." Every alteration to Hind's dates that we have found brings the results into closer accord with Ångström's curve, a fact which renders its failure for the next return more surprising. We propose examining this question further in a subsequent paper.

The observations of 607 are in a decided tangle; it seems clear that they refer to, at least, two different comets (some say that no less than four appeared in this year). Halley's comet seems to have appeared in the spring, and the date March 20 was selected for the purpose of computing the perturbations.

Revolution 607–684.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	$0°$–$30°$	$+''\cdot001$	$''$...	$+''28$
,,	330–360	$-\cdot005$	...	...
Earth	0–30	$-\cdot007$	...	-196
,,	330–360	$+\cdot003$	...	...
Jupiter	0–90	$-\cdot189$	-452	-5954
,,	90–270	$-\cdot126$	...	$+4877$
,,	270–360	$+\cdot491$	-157	-122
Saturn	0–90	$-\cdot051$	-37	-1438
,,	90–270	$+\cdot062$	...	$+2044$
,,	270–360	$-\cdot185$	...	$+30$
Uranus	0–360	$+\cdot006$	...	$+80$
	Sums	$-\cdot000$	...	-651

Taking n in 684 as $45''\cdot737$, n in $607 = 45''\cdot737$, and period in days $= \dfrac{1296000 + 651}{45\cdot737} = 28350$ days. Subtracting this from 1971199 (the adopted Julian day in 684) we obtain J.D. $1942849 = 607$ March 26.

At this point we have somewhat varied the former procedure; owing to the difficulty of deciding from observation the exact day of perihelion in 607, we took the well-established return of 451 July $3\cdot5 =$ J.D. $1885969\cdot5$ as a starting-point and assumed in a preliminary computation that the subsequent passage occurred in 530 November, in accordance with Hind. The verification of the date 530 November makes the revolution 451–530 the longest on record; it is about three months longer than 1066–1145 or 1222–1301,

which are very nearly equal in length. The adopted date is 530 November 15 = J.D. 1914959.

On this basis the perturbations were carried forward to 607, and it was found that nearly the same date was arrived at as in the previous backward reckoning. The value obtained for n in 607 differs somewhat, but this is easily accounted for when we note that in the three consecutive returns of 530, 607, 684 the observations are so vague that there is an uncertainty of a fortnight or so in the date of perihelion in each case. It is sufficient to know that our comet is correctly identified in each year, and we must be content with this small uncertainty in the dates.

Perturbations for the two revolutions 451–530, 530–607.

Planet.	Limits of u.	Revolution 451–530. $\int dn$.	$\int d\varpi$.	$\int d\zeta$.	Revolution 530–607. $\int dn$.	$\int d\varpi$.	$\int d\zeta$.
	° °	″	″	″	″		″
Venus	0– 30	+ ·018	...	+ 504	+ ·020	...	+ 560
,,	330–360	− ·013	...	...	− ·007	...	...
Earth	0– 30	+ ·003	...	+ 84	+ ·014	...	+ 392
,,	330–360	+ ·003	...	...	− ·011	...	...
Jupiter	0– 90	− ·2455	− 213	− 7179	+ 1·106	+ 633	+ 30974
,,	90–270	− ·0134	− 189	+ 7852	+ ·222	− 348	+ 1969
,,	270–360	+ ·6501	− 240	− 164	− ·734	− 295	+ 193
Saturn	0– 90	− ·0077	+ 56	− 223	+ ·196	− 18	+ 5498
,,	90–270	+ ·0248	− 101	+ 49	− ·073	− 10	− 237
,,	270–360	− ·0047	+ 4	+ 10	+ ·056	− 20	− 10
Uranus	0–360	− ·001	...	+ 17	+ ·002	...	− 15
	Sums	+ ·425	− 683	+ 950	+ ·791	− 58	+ 39324

For the revolution 451–530 we have the equation $\frac{1296000 - 950}{n \text{ at } 451} = 28990$, the observed period in days. Hence n at 451 $= 44''{\cdot}672$, n at 530 $= 45''{\cdot}097$.

For the revolution 530–607 we have the equation $\frac{1296000 - 39324}{45{\cdot}097} =$ period in days $= 27866$. Adding this to J.D. 1914959 we obtain J.D. 1942825 = 607 March 2, and n in 607 = $45''{\cdot}097 + ''{\cdot}791 = 45''{\cdot}888$. As the actual date of perihelion in 607 was probably at the end of March, we should have to diminish n in 530, 607 by about $0''{\cdot}05$, which would make a difference of only $0''{\cdot}09$ between the value of n in 607 found by forward reckoning, and that already reached by backward reckoning.

In carrying the research still further back, we have changed our method at this point, and used only the approximate tables given on p. 458 of the present volume, combined with the definite integral table on p. 178. This method is less accurate than that hitherto followed, and we must be prepared for errors of two or

three months in the computed periodic time, corresponding to 0″·15 or 0″·20 in the value of n; but this is near enough to test the accuracy of Hind's dates, which may be taken as presumably right if they fall within two months or so of the time indicated by our method; and, as a matter of fact, the results establish the accuracy of all Hind's dates from A.D. 451 back to B.C. 12, which is a very satisfactory conclusion. In his whole series there were only four errors, viz. A.D. 1223 (eleven months too late), A.D. 912 (four months too early), A.D. 837 (one month too late), A.D. 608 (one and a half years too late).

The research has been carried beyond the limits of Hind's list, and one fairly certain return has been added, that of B.C. 87. The description in Williams is, "In the second year of the epoch How Yuen (*i.e.* B.C. 87), the seventh moon (August), there was a comet in the east." Perihelion would be B.C. 87 August or September. Calculation indicates B.C. 163 June for the preceding passage, but no definite observation can be found in this year; Pingré has several vague references to comets about this epoch, but they are so wanting in precision that no use can be made of them. Going back another round, the date B.C. 239 January was found; we think it not unlikely that the comet observed in the spring of B.C. 240 was Halley's.

The discordance from our date is not greater than we might reasonably expect when the approximate method is used without check for two revolutions, and the characteristics of the comet of B.C. 240 are very like those exhibited by Halley's comet when perihelion falls about April.

Williams says, "In the seventh year of the reign of Che Hwang (B.C. 240) a comet first appeared in the east. It was afterwards seen in the north. In the fifth moon (May) it was seen for 16 days in the west."

For the two preceding passages there are no comets in either Pingré or Williams that could possibly be Halley's. Three revolutions earlier, there is the following in Williams: "In the second year of the Emperor Ching Ting Wang (B.C. 467) a comet was seen." This is at about the time when we should expect Halley's comet, but the identity cannot become more than a vague conjecture.

To sum up, we have carried the comet with fair certainty back to B.C. 87, with some probability back to B.C. 240; at this point we are brought to a standstill by the complete absence of earlier observational material.

The perturbations for the different rounds are given in the following tables. In each revolution $\dfrac{1296000 - \int d\zeta}{n \text{ at beginning of rev.}} =$ period in days; an equation which gives n when the period is assumed, or which gives the period when the value of n at the end of the period is assumed, and the value at the beginning deduced by applying the calculated perturbation of n.

Dates of beg. and end of Rev.		373 November 7; 451 July 3			295 April 7; 373 November 7			218 April 6; 373 April 7		
Values of g' for $u=$ 0°, 90°, 270°, 360° } Jupiter / Saturn		352°, 218°, 322°, 189° / 247, 338, 25, 116			126°, 355°, 123°, 352° / 7, 99, 155, 247			309°, 173°, 262°, 126° / 146, 236, 277, 7		
		$\int dn.$	$\int d\varpi.$	$\int d\zeta.$	$\int dn.$	$\int d\varpi.$	$\int d\zeta.$	$\int dn.$	$\int d\varpi.$	$\int d\zeta.$
Jupiter	0° – 90°	+″·770	−290″	+21972″	−″·120	+258″	−3539″	+″·330	−583″	+9369″
,,	90 – 270	−·074	−242	−3697	+·103	−162	+7862	−·249	−163	−2282
,,	270 – 360	−1·040	+190	+168	+·210	−500	−45	−1·270	+700	−6
Saturn	0 – 90	−·02	−77	−567	+·150	−20	+4330	−·036	+55	−1018
,,	90 – 270	+·037	+50	+2142	−·059	−31	−280	+·047	−110	+608
,,	270 – 360	−·110	−66	+20	+·060	+17	−10	−·050	+3	+10
	Sums	−·437	−435	+20035	+·344	−438	+8318	−1·228	−98	+6681

Dates of beg. and end of Rev.		141 March 25, 218 April 6			66 January 26, 141 March 25			B.C. 12 Oct. 8, A.D. 66 Jan. 26			B.C. 87 Aug. 15, B.C. 12 Oct. 8		
Values of g' for $u=$ 0°, 90°, 270°, 360° } Jupiter / Saturn		132°, 355°, 85°, 309° / 285, 15, 56, 146			12°, 231°, 274°, 132° / 86, 174, 197, 285			217°, 79°, 150°, 12° / 234, 324, 357, 86			95°, 314°, 358°, 217° / 36, 124, 146, 234		
		$\int dn.$	$\int d\varpi.$	$\int d\zeta.$	$\int dn.$	$\int d\varpi.$	$\int d\zeta.$	$\int dn.$	$\int d\varpi.$	$\int d\zeta.$	$\int dn.$	$\int d\varpi.$	$\int d\zeta.$
Jupiter	0° – 90°	−″·190	+200″	−5459″	+″·940	−114″	+25968″	−″·217	−373″	−6128″	+″·848	+640″	+23305″
,,	90 – 270	+·244	−68	+7788	−·224	−464	−3417	−·018	+246	+6584	+·269	−492	+3568
,,	270 – 360	−·047	−600	+43	−1·280	+710	+23	+·328	−447	−94	−·853	−97	+197
Saturn	0 – 90	+·055	−95	+1578	+·010	+16	+285	−·036	−58	−1003	+·058	−54	+1606
,,	90 – 270	−·005	+78	+1630	−·034	−129	−462	+·063	+33	+2142	−·075	−64	−596
,,	270 – 360	−·038	−86	+4	+·035	+50	−10	−·192	−22	+30	+·060	0	−10
	Sums	+·019	−571	+5584	−·573	+69	+22387	−·072	−621	+1531	+·307	−67	+28070

Dates of beg. and end. of Rev.			B.C. 163 May 20; B.C. 87 Aug. 15			B.C. 240 May 15; B.C. 163 May 20		
Values of g' for $u=$ 0°, 90°, 270°, 360°	Jupiter / Saturn		302°, 163°, 233°, 95° / 185, 274, 307, 36			125°, 349°, 78°, 302° / 323, 53, 94, 184		
			$\int dn.$	$\int d\varpi.$	$\int d\zeta.$	$\int dn.$	$\int d\varpi.$	$\int d\zeta.$
Jupiter	0°	90°	+″·260	−607″	+7321″	−″·090	+280″	−2596″
,,	90	270	−·307	−63	−1561	+·254	−89	+7326
,,	270	360	+·054	+15	−120	−·093	−600	+62
Saturn	0	90	−·060	+15	−1691	+·163	−73	+4569
,,	90	270	+·074	−66	+1484	−·049	+52	+704
,,	270	360	−·174	+16	+22	+·018	−76	−24
		Sums	−·153	−690	+5455	+.203	−506	+10041

The date B.C. 240 May 15 was deduced from observation, not from calculation. The date calculated is B.C. 239 Jan. If we suppose an error of two months in the same direction in each of the calculations of the last two revolutions (which is quite probable), the discordance would almost disappear. It appears worth while to calculate the three revolutions B.C. 12 to B.C. 240 by more exact methods, and we hope to undertake this at a later date.

The following table gives the values of n calculated from the above results for the different revolutions, and also smoothed values resulting from the combination of values from consecutive revolutions.

Perihelion Passage.	n.	Smoothed n.
A.D. 684 / 607	45″·737 / 45·747	45″·742
607 / 530	45·888 / 45·097	45·818
530 / 451	45·097 / 44·672	45·085
451 / 373	44·54 / 44·98	44·604
373 / 295	45·19 / 44·845	45·095
295 / A.D. 218	44·617 / 45·845	44·715

Perihelion Passage.	n.	Smoothed n.
A.D. 218 / 141	45″·88 / 45·86	45″·862
141 / 66	45·817 / 46·39	45·835
A.D. 66 / B.C. 12	46·398 / ·46·47	46·397
12 / 87	46·497 / 46·19	46·483
87 / B.C. 163	46·185 / 46·34	46·185

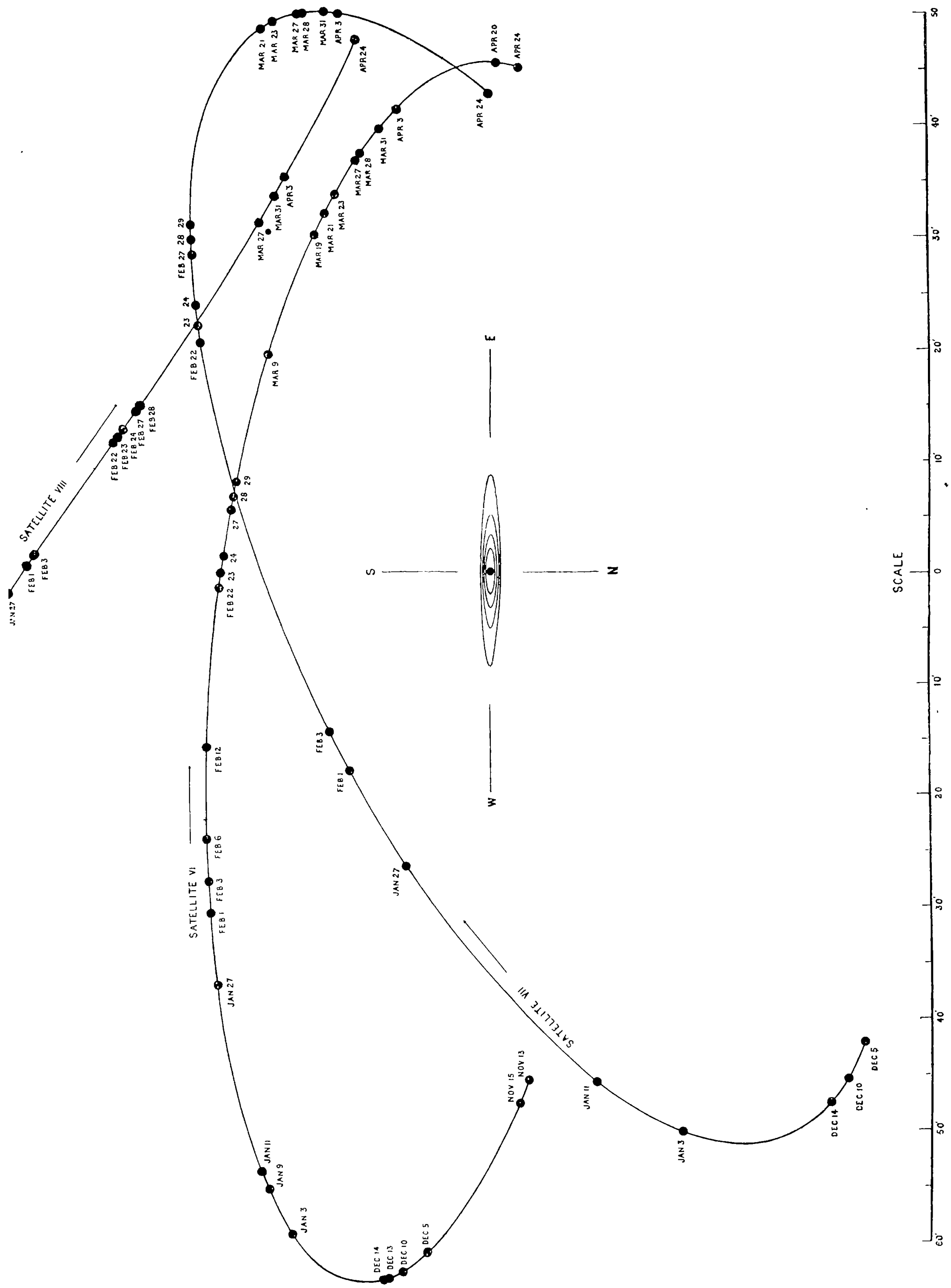

Diagram showing the positions of Jupiter's Satellites VI, VII, and VIII, from photographs taken at the Royal Observatory, Greenwich, during the Opposition of 1907–8.

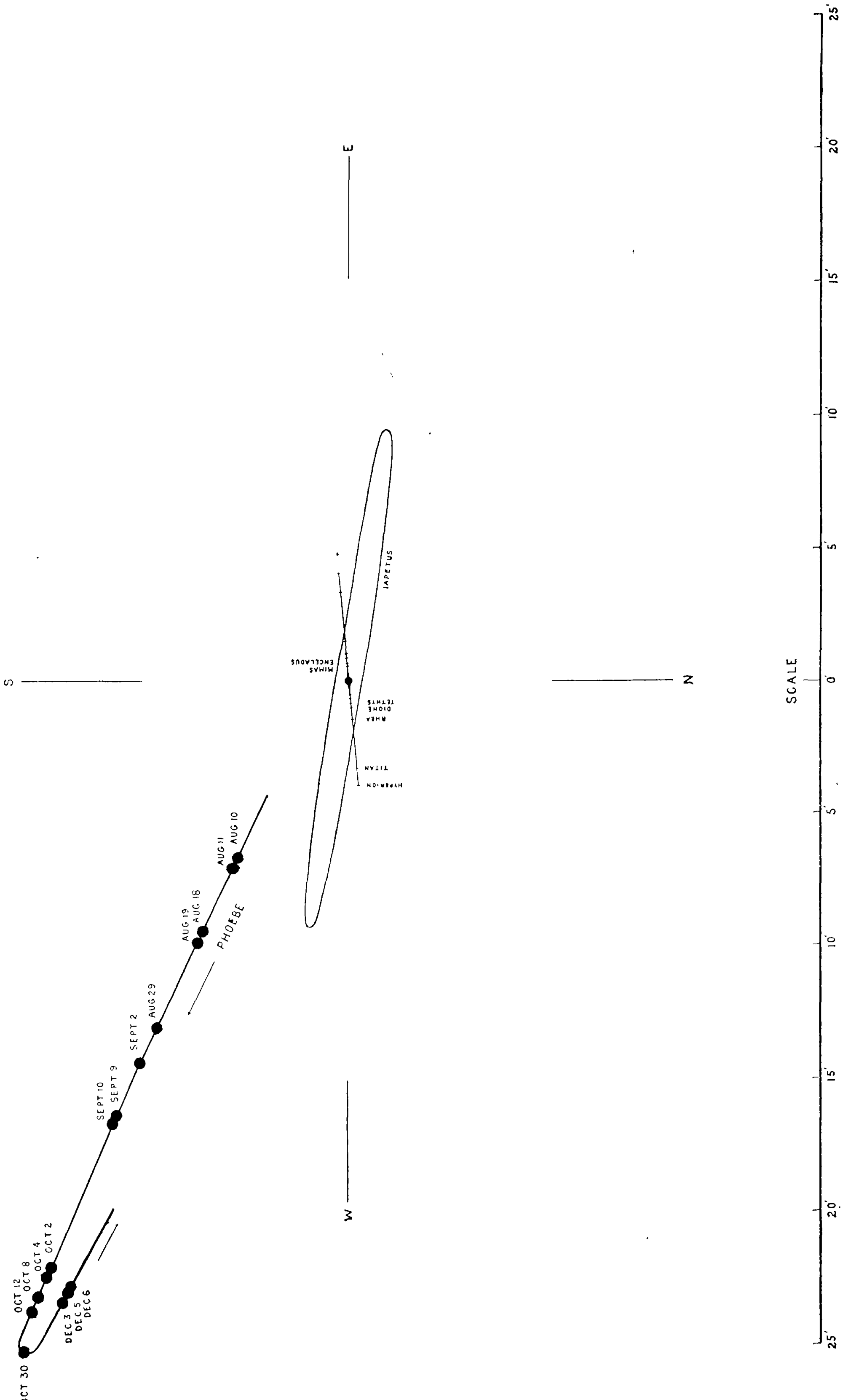

Diagram showing the position of Saturn's Ninth Satellite—Phœbe, from photographs taken at the Royal Observatory, Greenwich, during the Opposition of 1907.